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LOCALIZED STRESSES COMBINED WITH GEOMETRIC DISCONTINUITIES: STUDY OF A SHAFT.

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ABSTRACT

Due to its functional characteristics, machine components are designed with various geometrical discontinuities, usually combined in a single section or sections very close. Localized stresses they generate are difficult to obtain if not by computer. The classical literature offers no stress concentration factors for cases as common as those seen in, for example, the design of a shaft, where coexist, with various geometric discontinuities, efforts as torque and bending combined. In this work we present and analyze the results obtained from the Von Mises stresses occurring in areas of a shaft with change of diameter and a keyway, considering the load applied as a lateral pressure from the key generated to transmit a given torque. It is also considered a bending effort. The objective is to obtain the Von Mises stresses values for different positions of the keyway with respect the fillet radius for the change of diameters and also to found the influence of the proximity of the keyway on the values of the stresses in the fillet radius.

INTRODUCTION

In the mechanical design of machine components, it is common to include various geometric discontinuities so that the component complies with the utility for which it is designed. It is very likely that these discontinuities are superimposed in the same resistant section or sections that are located very close together. The interaction that occurs results in the generation of localized stresses whose values are difficult to obtain except by means of tests or computer tools.

In a review of the state of the art [1] it is possible to observe that the available literature specialized in mechanical design [2,3,4], metal fatigue [5], or that where it is possible to find values of the concentration factor [6.7], there is no information about the particular treatment of mechanical parts combined with geometric discontinuities in complex stress states. Leaving aside the studies on flat plates with several holes, distributed in different ways, well studied by several authors [8-15], with respect to non-flat pieces, there is little data available and this is limited to be applied to particular cases and specific designs [16-18].

Despite the existence of software that calculates the stresses in machine elements for any complex shape and charge state, it is of fundamental importance to optimize the productivity of the work and obtain reliable values with the appropriate pre-design concept and how to present the state of charge for analysis.

In this work it is presented and analyzed results obtained in the area of a shaft, which combines a change in diameter and a flat keyway. It is applied to the shaft a complex state of stress due to the existence of torque and bending. There are obtained different values of the Von Mises stresses, depending on the distance from the slot of the keyway with respect of the fillet radius between the two diameters, which are compared with results obtained by this research team, in a previous work [16], but basically, it will result evident the importance of how the torque is applied.

OBJECT OF STUDY

The object of study is the sector of a shaft with a change in the diameter and keyway in the minor section, as shown in Figure 1. There are determined the stresses in the fillet area and in the keyway.

The acting loads are: torque (Mt) and bending (Mf). The analysis variable is the distance "*j*" measured from the keyway end to the fillet radius “*r”* between diameters *“D”* and *“d”*, as shown in Figure 3.

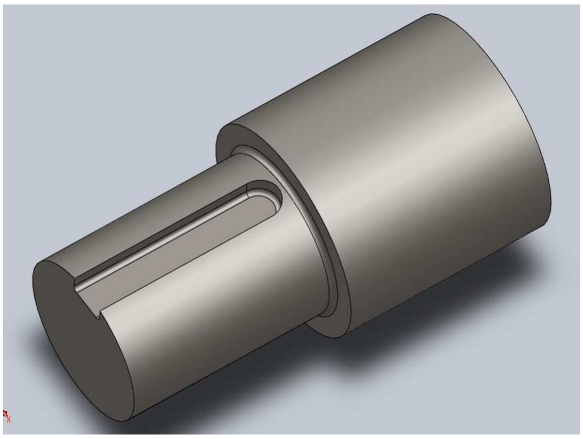

Figure 1 Object of Study

Figure 2 shows the dimensions of the shaft part which is under study, which are: D = 99.75 mm diameter, minor diameter d = 75.00 mm, D / d = 1.33 (to match Peterson [7] Page 76, Fig. 67), fillet radius r = 2.50 mm (taken from a SKF bearing manual, to host a ball bearing Nº 6415), then it is obtained the ratio r / d = 2.50/75 = 0.0333.

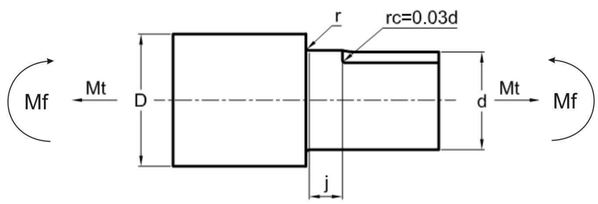

Figure 2 Loads applied and geometry of object of study

Figures 3 and 4 show longitudinal and transverse dimensions of the slot respectively, which (except length) remain unchanged during the analysis of stresses variation.
They are: depth t = d / 8 = 75/8 = 9.375 mm, width b = d / 4 = 18.75 mm (to match Peterson [7] p 118, Figure 100), ratio r / d = 0.03 (ibid.); so r =d x 0.03 = 75 x 0.03 = 2.25 mm (radius at the bottom of the keyway).

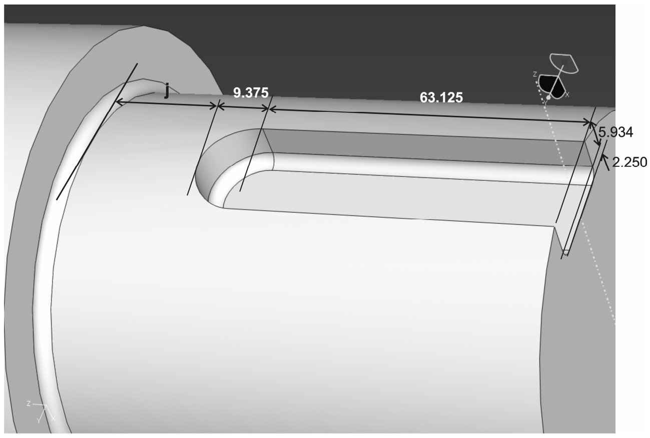

Figure 3 Longitudinal dimensions of the keyway.

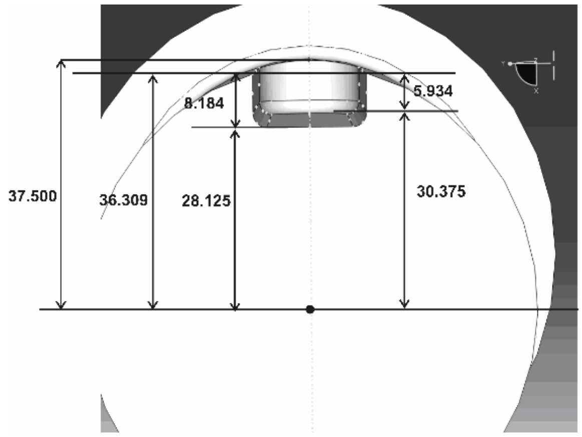

Figure 4 Transverse dimensions of the keyway

The Mt applied torque is imposed at 10,000,000 Nmm and flexion Mf at 1,000,000 Nmm.

The value of the torque determines the lateral pressure “*p”* generated by the key on the longitudinal side of the keyway. It is considered useful for this purpose, only the height of 5.934 mm (it is understood that cannot exists pressure on the radius of the bottom of keyway). The length of the slot will vary depending on *“j”*. It is subtracted from the total length -Figure 3 - the keyway end radius, 9.375 mm.

In this way of presenting the principal load, derived from the torque, lies the originality of this analysis with respect to which consider loads applied in a concentrated form and in theory at the ends of the shaft.
Can be written:

$$p = \frac{Mt}{S \cdot y} \qquad (1)$$

Where:

S S is the section of the lateral longitudinal face of the keyway with a height of 5.934 mm and its values is obtained from:

$$S = 5{,}934 \cdot (l - j - 9{,}375) \qquad (2)$$

l = 97,5 mm, the length of the shaft section with diameter d = 75 mm.

y is the distance from the section center to half of the height of the lateral longitudinal face of the keyway and equal to 30.375 + (5.934 / 2) = 33.342 mm.

Due to that the keyway length is variable, and to maintain constant torque value, the pressure *"p"* on the lateral longitudinal face of the keyway must be variable. Different values of this pressure are given in the titles of paragraphs a, b and c.

METHODOLOGY AND RESULTS

It is defined a steel with a Poisson ratio: 0.3 and longitudinal elastic modulus of 210 000 MPa.

For modeling it was considered a third-degree link (embedded) for the end of larger diameter, assuming that this side is output power.

In order to obtain the theoretical stress concentration factor Kt, through the relationship between actual values of the stresses in the areas of analysis and the theoretical, it was calculated, in traditional manner, Von Mises nominal stress, for full section of diameter d = 75mm, with values of torque and bending above indicated.

It was used this expression:

$$\sigma_{VM} = (\sigma^2 + 3 \cdot \tau^2)^{1/2} \qquad (3)$$

Where:

σ Normal stress due to bending $= Mf/W = 32Mf/\pi d^3 = 24{,}14\ N/mm^2$

τ Shear stress by torsion $= Mt/Wp = 16Mt/\pi d^3 = 120{,}72\ N/mm^2$

And the nominal Von Mises stress is:

$$\sigma_{VM} = 210{,}48\, N/mm^2 \qquad (4)$$

The Von Mises maximum stresses were obtained by means of a Finite Element Analysis software. This software have been already validated en previous studies [8] and by other authors [13, 19, 20], who agree in the usefulness of this tool and also they have obtained good correlation between mathematical and experimental results. These authors have been realized a convergence study which leads to the conclusion that with a triangular mesh with 0.23 mm side (average), stresses values are stabilized.

Thus, Von Mises maximum stresses were obtained, located in the areas shown in Figures 5 to 10, for j = 25 mm, j= 10 mm and j = 0. The software worked with adaptive meshing and automatic iterations.

These results are then related with that obtained in (4) to determine the stress concentration factor in each of the points of interest.

a- Von Mises Stresses for j = 25 mm (*p* = 801 MPa).

For the keyway, the maximum stress of 2232 MPa is recorded. For the area of fillet radius between diameters, the maximum stress obtained is 1294 MPa.
Figures 5 and 6 show the graphs obtained from machine output and location of areas where the values were obtained.

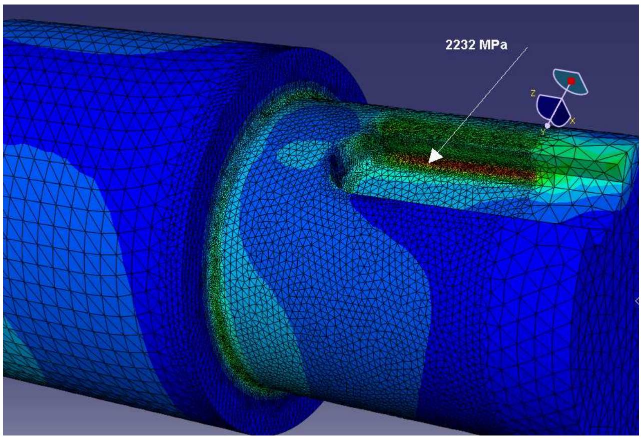

Figure 5. Value of maximum Von Mises stress in the keyway, for j = 25 mm.

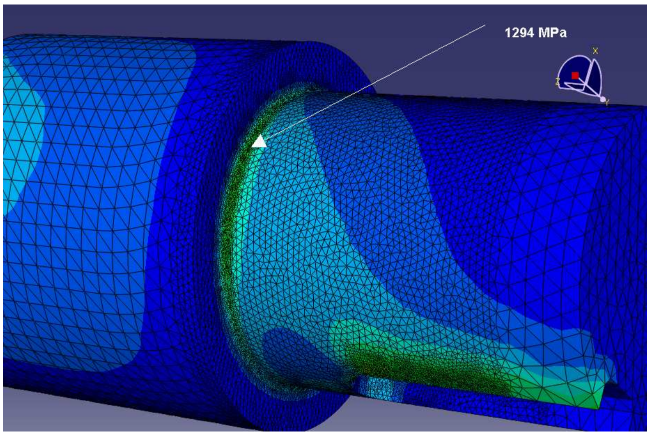

Figure 6. Value of maximum Von Mises stress in fillet radius area, for j = 25 mm.

b-Von Mises Stress for *j* = 10 mm (*p* = 647 MPa)

For the keyway, the maximum stress of 1768 MPa is recorded. For the area of fillet radius between diameters, the maximum stress obtained is 1163 MPa.
Figures 7 and 8 show the graphs obtained from machine output and location of areas where the values were obtained.
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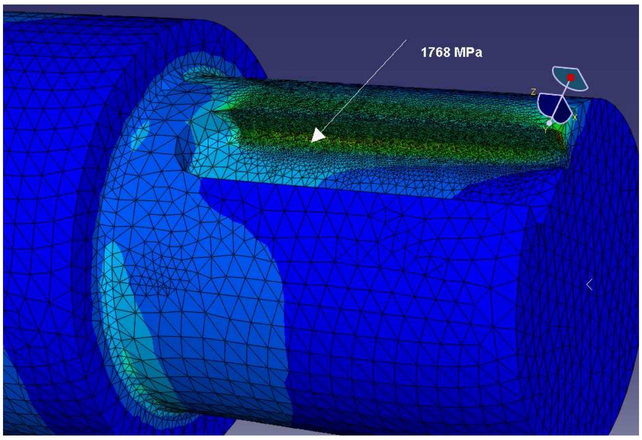

Figure 7. Value of maximum Von Mises stress in the keyway, for j = 10 mm.

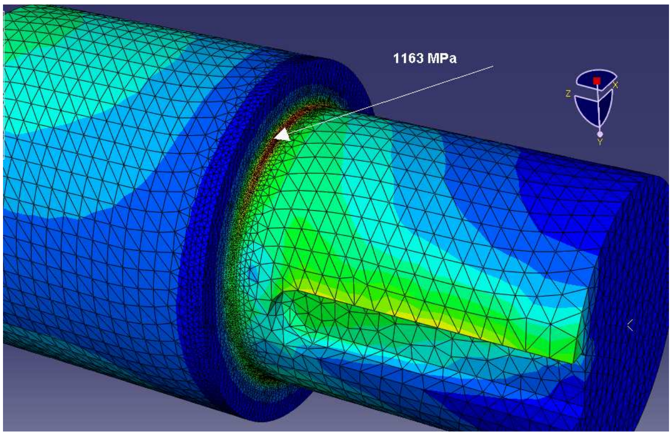

Figure 8. Value of maximum Von Mises stress in fillet radius area, for j = 10 mm.

c-Von Mises Stress for j = 0 mm (p = 573 MPa).

For the keyway, the maximum stress obtained is 1598 MPa. For the fillet radius area, the maximum stress recorded is 1105 MPa. Figures 9 and 10 show the graphs obtained from machine output and location of areas where the values were obtained.

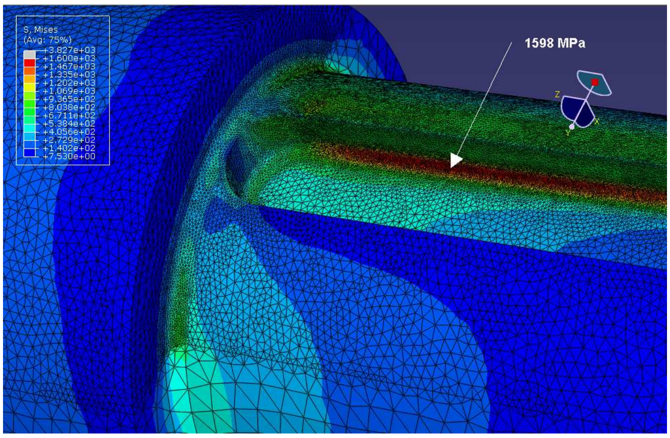

Figure 9. Value of maximum Von Mises stress in the keyway, for j = 0 mm.

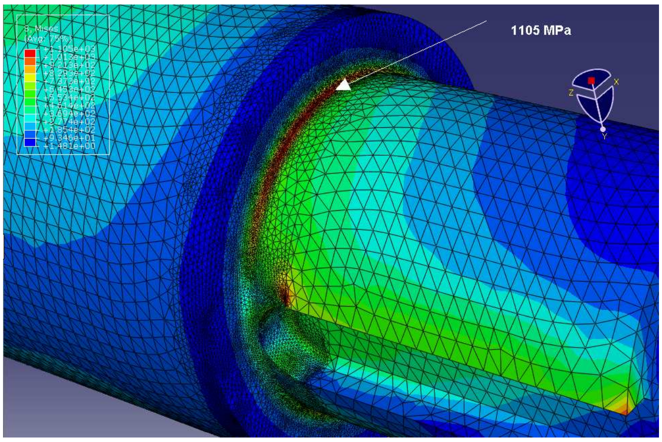

Figure 10. Value of maximum Von Mises stress in fillet radius area, for j = 0 mm.

d-Summary table of results.

Table 1 shows the values obtained and the stress concentration factor resultant.

Table 1. Von Mises Stress and stress concentration factor
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j (mm)	Von Mises stress (MPa)		K_t Factor	
	Keyway	Fillet radius	Keyway	Fillet radius
0	1598	1105	7,59	5,24
10	1768	1163	8,40	5,52
25	2232	1294	10,60	6,15

Note the significant decrease in keyway stress as *"j"* tends to 0. This is attributed to the greater length of the keyway. The same applies to the fillet radius between diameters, but to a lesser extent, indicating that the overlap between both discontinuities has a low negative impact.

CONCLUSIONS

The simplified way of modeling the application of the moments acting in the manner commonly presented in the literature and applied in a previous work [16] of these authors, induces low levels of stresses and almost no differences between areas under study. However, in this analysis, there are strong differences between the stresses of the keyway and the fillet radius for any of the distances *"j"*. Note that for $j = 0$ mm, the relationships between the stress at keyway area with respect to fillet radius is 1.45 times, in case of $j = 10$ mm 1.52 times and for $j = 25$ mm 1.72 times.

The stress in the keyway decreases for $j = 0$, which is due to the reduced pressure on the longitudinal side of the keyway (with equal time transmitted torque). This is valid as a design strategy, because the increase of keyway length decreases the stresses generated. However, it is far superior to those obtained in the fillet radius. It states that it is more significant the keyway as concentrator, even coinciding with the zone of fillet radius between the two diameters. Importantly, with respect to the previous study [16], the location of the maximum stress in the keyway, always in the range of fillet radius between its lateral side and the bottom of it, has moved from a position coincident with the curved end to the middle of the length of the slot. This may be due to the criterion used to define the pressure, because said curved area has been ignored by their inability to transmit torque.

The maximum tension recorded for the fillet radius is located approximately 90° with respect to the keyway, and also decreases as *"j"* does. This situation, plus the fact of not being affected by the proximity of the keyway, is significantly different from the results obtained in the previous study [16]. The authors believe that this result is due to the bending moment generated by the lateral pressure in the keyway, value significantly higher than Mf = 1,000,000 Nm, considered as external load.

The manner as the torque is applied in this work, have a significant impact on results. Considering the pressure exerted by the key, as the way to transmit the torque on the longitudinal face of the keyway, is closer to reality than if we consider the applied loads as punctual loads.

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