Two Ways of Modeling Efforts and the Combined Influence of Geometry on the Located Stresses in a Shaft in Bending and Torsion

Ricardo Mario Ame [1], and Gabriel María Dasso [2]

Abstract: **In the classical literature offers no stress concentration factors for cases as common as those seen in, for example, the design of a shaft, which coexist with various geometric discontinuities efforts torque and bending combined. In this work we present and analyze the results obtained from the Von Mises stresses occurring at the end of a shaft with diameter and keyway change plan, considering the loads applied to the lateral pressure from the key generating to transmit a given torque. The objective is to obtain the values for different positions of the slot on the radius of agreement between the diameters found the influence of the proximity of the slot on the values of the stresses in the fillet radius of the diameters and no action on tensions in the bottom of the keyway.**

Keywords: Localized Stresses; Mechanical Design

1. INTRODUCTION

In the mechanical design of transmission shafts must include various geometrical discontinuities to its compliance with the utility for which it built. It is very likely that these discontinuities overlap in the same resistant section or sections are located close together. The interaction that takes place is unexpected and varied and may result in the generation of high value localized stresses which calculation is difficult. In opportunities necessitates testing scale models or to apply software tools. The greater reliability in the results you get will depend on a proper modeling of the forces acting.

In a review of the state of the art [1] you can check the available literature specializing in mechanical design metal fatigue or indicating that the values of stress concentration factor, does not provide information on how to obtain them for shafts with combined geometric discontinuities in complex states of stress. Niemann [2] the partially exposed and incomplete; Forrest [3] as close to what we're dealing with, you can read on pages 327 and subsequent which presents the work of Gough related to the combined efforts of bending and torsion in the presence of only keyways. For his part, Faires [4] addresses the issue of hubs combined in a very generic, offering a tip of application of dubious reliability; Deutschman et al. [5] referring to the possibility of meeting with two stress concentrators, says: "*The limited information available indicates that the cumulative results of the two factors are greater than each of the individual factors but less than the product of two factors . However, Lipson and Juvinall suggest you use the product of two factors theoretical stress concentration in the notch sensitivity equation for Kf and Kfs* ". Reading these authors, Lipson and Juvinall [6] indicate, in Chapter 11 of Part II: "In many cases, members of machines do not contain a simple stresses hub but many discontinuities such as blood types are involved.

The resulting concentration factor of the total number of notches as large cannot be on individual concentration factors due to the coupling effect of the stresses caused by the link between stress concentrators ", and finally, after proposing making the product of individual factors, suggest out tests because their results may shed less than the calculated value. Although this reference material suffers from the ancient times, other contemporary authors [7-9] do not address. In literature with abundant information on stress concentrators [10-11] the information is not considered part combinations of geometric discontinuities. The availability of data is limited to particular cases applied to specific designs [12-16] but there is no practical use generic information.

This paper presents and analyzes the results obtained in the area of a shaft, whose design combines a change in diameter and a flat keyway. The existence of a moment of torsion and bending other complex generates a state of stresses. Different values are obtained from the Von Mises stresses in two zones: the bottom of the keyway and the end of the fillet radius smaller diameter measures both for different distances between the edge of the keyway slot and the fillet radius between two diameters. These results also are compared with those obtained by the same team of researchers in a previous work [17] and which are determined by the current use abacuses [10-11]. In addition to analyzing the incidence localized stresses mutual between both geometric discontinuities, discusses the importance of modeling efforts applied.

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2. THE OBJECT OF STUDY

The object of study is the sector of a shaft with diameter change and keyway in the lower section, as shown in Figure 1. Determine the stresses in the area of the fillet and in the keyway. The applied loads are torsional moment Mt and Mf bending moment (this acting in the vertical plane passing through the center of the keyway slot, with the meaning as indicated in Figure 1). The analysis variable j is the distance measured between the end of the keyway and the end of the fillet radius r between both diameters D and d, as shown in Figures 2 and 3.

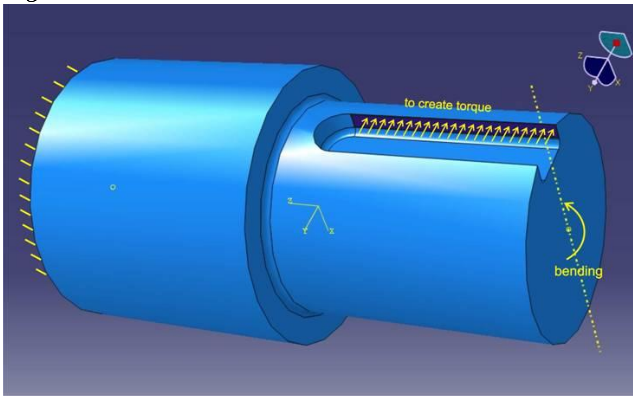

Fig. 1. The object of study.

Figure 2 shows the dimensions of the shaft section under consideration, which are major diameter D = 99.75mm, smaller diameter d = 75.00 mm, D / d = 1.33 (to approximate Pilkey relations given in [11] 164 pages page 166 Chart 3.10 and Chart 3.12), radius r = 2.50 mm (taken from a manual SKF bearing host a radio to 6415), then it is the ratio r / d = 2.50/75 = 0.0333.

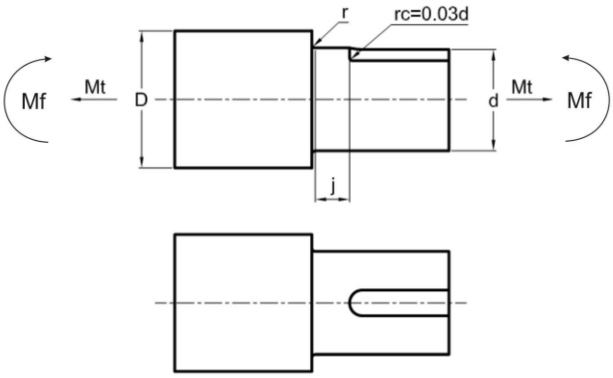

Fig. 2. Condition of loading and geometry of the object of study.

In Figures 3 and 4 show respectively longitudinal and transverse dimensions of the keyway, which (except the length of which will vary inversely with the level j) remain unchanged during the analysis to be performed.

They are: depth t=d/8=75/8=9.375 mm, width b=d/4=18.75 mm (to match Pilkey [10] Page 408 and 409, Chart 5.1 and Chart 5.2), ratio r/d=0.03 (ibid.) as r=d×0.03=75×0.03=2.25 mm (radius at the bottom of the slot).

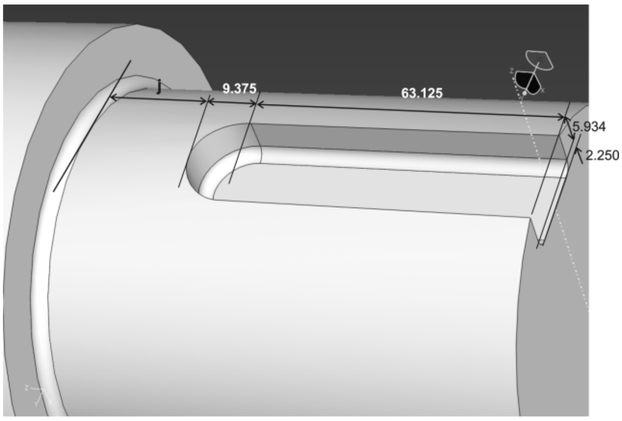

Fig. 3. Dimensions longitudinal keyway.

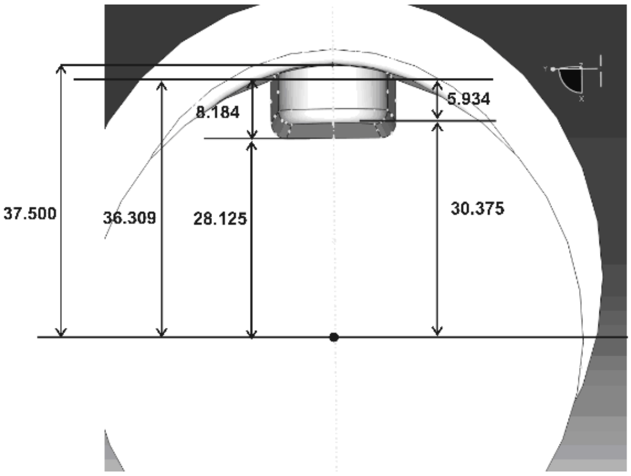

Fig. 4. transverse dimensions of the keyway.

The applied torque is imposed Mt 10,000,000 Nmm and 1,000,000 Nmm Mf moment bending.

The torque value to determine the lateral pressure p generated by the key on the longitudinal side of the slot (Figure 1), considered useful for this purpose, only the height of 5.934 mm (it is understood that there can be no uniform pressure on the keyway bottom radius). The length of the slot is variable depending on j. Subtracting the total length 9.375 mm -radius Figure 3- in the starting curve keyway.

This way of showing the action of torque makes the difference between this analysis and the work done in [17]. In that study, the torque applied it was considered timely to the far right of the tree, as a concentrated load, whereas in the present work, is the result of pressure p as distributed load acting on the side of the keyway .

Can be written

$$p = \frac{Mt}{S \cdot y} \quad (1)$$

Where:

S: is the section of the lateral longitudinal face of the slot height 5.934 mm and responds to:

$$S = 5{,}934 \cdot (l - j - 9{,}375) \tag{2}$$

Where:

l: 97.5 mm, the length of the shaft section of diameter d = 75mm.

y: is the distance from the center of the middle section of the height of the longitudinal lateral face of the keyway and equal to 30.375 + (5.934 / 2) = 33.342 mm.

Turns out to be variable length of the keyway, and to maintain constant torque value, the pressure *p* on the lateral longitudinal face of the keyway must be variable. The different values of this pressure is entered in the titles of paragraphs a, b and c

3. METHODOLOGY AND RESULTS

For modeling is considered a link of third degree of the larger diameter end (Figure 1), a steel with a Poisson ratio: 0.3 and longitudinal elastic modulus of 210 000 MPa.

In order to obtain the theoretical stress concentration factor Kt, by the ratio of the actual values of the stresses in the areas of analysis and theoretical, calculated, traditionally, the stresses of Von Mises, for section full diameter d = 75mm, with torque values and the previously mentioned bending:

We used the expression:

$$\sigma_{VM} = (\sigma^2 + 3 \cdot \tau^2)^{1/2} \tag{3}$$

Where:

σ: Normal stress for bending moment and equal to $Mf/W = 32Mf/\pi d^3 = 24{,}14\ N/mm^2$

τ: shear stress for torsional moment and equal to $Mt/Wp = 16Mt/\pi d^3 = 120{,}72\ N/mm^2$

And the nominal Von Mises stress is:

$$\sigma_{VM} = 210{,}48 N/mm^2 \tag{4}$$

Using software finite element stress analysis the maximum stresses Von Mises obtained from the shaft. This software has already been validated in previous studies [18] and other authors [15-16, 19]. There is agreement as to the appropriateness of using this computational tool, since only small differences were obtained results with those obtained by mathematical analysis and testing.

For this study, the authors worked with and iterations adaptive meshing automatic convergence conducted a study to find the stabilization of the stresses values obtained for a triangular mesh size with sides, on average 0.23 mm measured.

The scale is usually indicated in graphics software that delivers only shown in Figures 5 and 6 as an example, and not placed in the other, because the mesh can not discern such small values from the colors displayed in it, which takes away their presence felt. It shows the maximum values and is supplied from the scale need.

Then these results relate to that obtained in (4) to determine the stress concentration factor at each of geometric areas of interest.

A. Von Mises stresses for j = 25 mm (p = 801 MPa).

Figure 5 is a graphical representation which provides the software and indicates the location and the maximum voltage value registered in the keyway which is 2232 MPa.

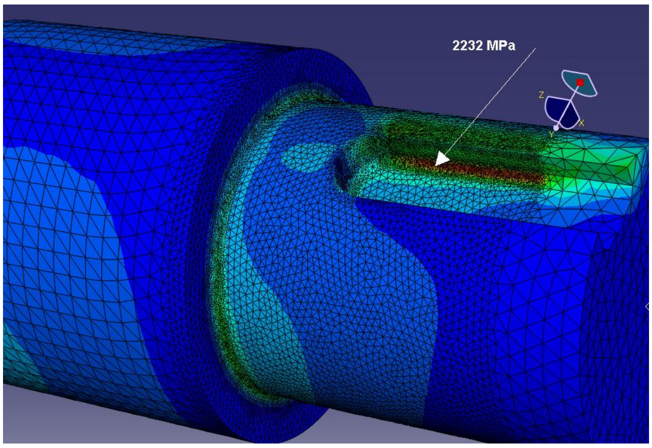

Fig. 5. Maximum stress value of Von Mises in the keyway, for j = 25 mm.

Figure 6 shows the location and the maximum value of the stress obtained in the fillet radius of 1294 Mpa.

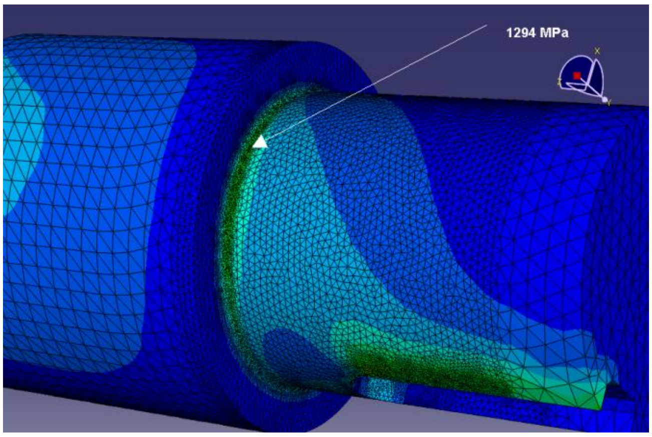

Fig. 6. Value of maximum Von Mises stress in the according radius, for j = 25 mm.

B. Von Mises stresses for j = 10 mm (p = 647 MPa)

Figure 7 shows the location and magnitude of the maximum stress located in the keyway of 1768MPA

According to the area between diameter, Figure 8 shows the maximum value of the stress obtained of 1163 Mpa and its location.

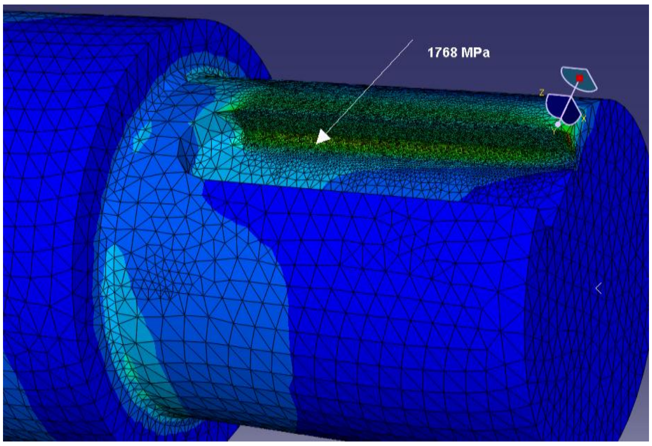

Fig. 7. Maximum stress value of Von Mises in the keyway, for j = 10 mm

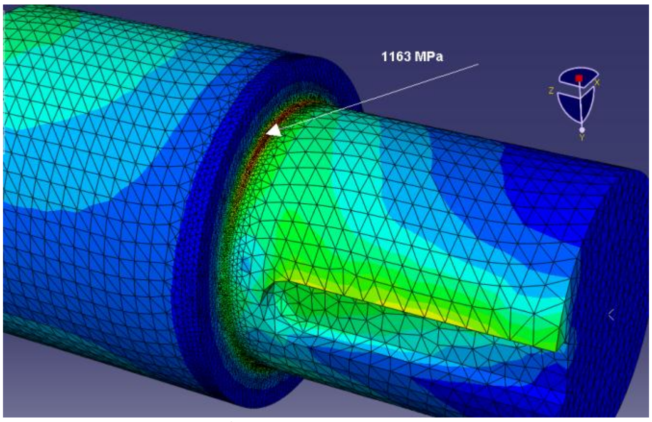

Fig. 8. Maximum value of Von Mises stress on the according radius to j = 10 mm.

C. Von Mises stresses for j = 0 mm (p = 573 MPa).

Figure 9 indicates the location and maximum stress value registered in the keyway: 1598 MPa.

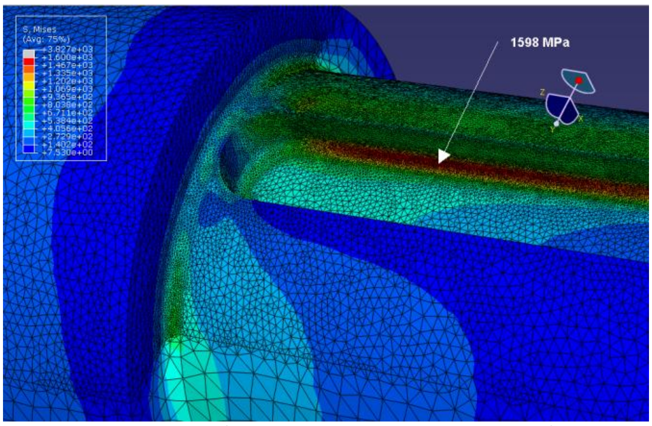

Fig. 9. Maximum value of the Von Mises stress in the keyway, for j = 0.

In accordance radius the maximum stress obtained is 1105MPa and its location shown in figure 10.

D. Summary table of results.

Table 1 shows the values obtained in this study and the stress concentration factor resulting.

TABLE 1.
VON MISES STRESSES AND STRESS CONCENTRATION FACTOR IN THIS STUDIO.

j (mm)	Obtained von Mises stress (MPA)		KT Factor	
	keyway	according radius	keyway	according radius
0	1598	1105	7,59	5,24
10	1768	1163	8,40	5,52
25	2232	1294	10,60	6,15

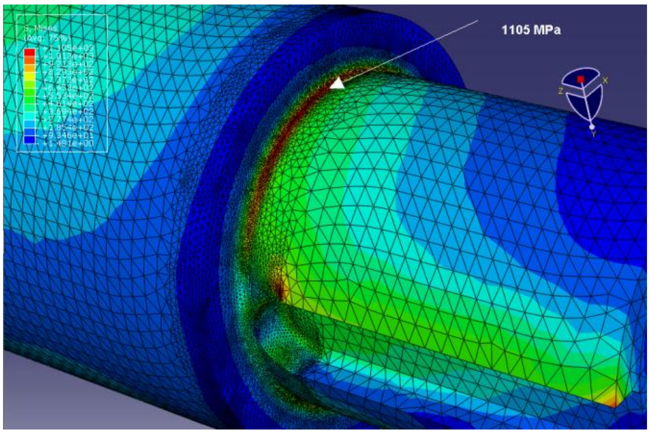

Fig. 10. Maximum value of the Von Mises stress in the according radius, for j = 0

Table 2 shows the results of the work [17] made previously.

TABLE 2.
VON MISES STRESSES AND STRESS CONCENTRATION FACTOR IN PREVIOUS STUDY [17].

j (mm)	Obtained von Mises stress (MPA)		KT Factor	
	keyway	according radius	keyway	according radius
0	398	397	1,89	1,89
10	396	383	1,88	1,82
25	396	375	1,88	1,78

Table 3 shows the stress concentration factors depending Pilkey graphs [10] and compared with those reported in Table 1.

TABLE 3.
STRESS CONCENTRATION FACTOR RESULTING FROM THIS STUDIO AND THEIR COMPARISON WITH THOSE GIVEN IN PILKEY [10]

j (mm)	According Pilkey		According to this study	
	keyway	according radius	keyway	according radius
0	1,95*	1,82*	7,59	5,24
	2,41**	2,37**		
10	1,95*	1,82*	8,40	5,52
	2,41**	2,37**		
25	1,95*	1,82*	10,60	6,15
	2,41**	2,37**		

Note: * for torsional moment; ** for bending moment

The comparison of the results shown in Tables 1 and 2 for the same distances j shows significant differences in the values of the stresses obtained according to the way of considering the applied loads. The simplified way of modelling the application of the moments acting in the manner commonly presented in the literature, and applied in the work [17], leads to obtaining low values of stress and stress concentration factors.

From the results according to the work [17], can also be observed discrepancy between the low stress values for different locations of the slot, given by the dimension j between the study areas.

Noting now the results included in Table 1 shows the significant reduction of tension in the keyway as j tends to 0. This is attributed to the greater length of it. Tensions also decrease in the agreement, but to a lesser extent, indicating the low incidence involving both geometrical discontinuities coincide.

The numbers shown in Table 3 can be deduced that avoids an explicit function leading to a predictable result from factors available in the literature. Note that for the keyway, for example, there is a combination to obtain the values of the stress concentration factor obtained by the analysis performed with the software.

4. CONCLUSIONS

The simplified way of modelling the application of moments acting the way they are commonly presented in the literature and applied in a previous work [17] of these authors, induces low levels of stress and almost no differences between study areas . This modelling effort implementing induce marked differences between strains of the keyway and the according radius for any of the distances j. Note that for j = 0, the relationship between the stress at point A with respect to B is 1.45 times, j = 10, 52 and j = 25 72 times.

The maximum stress recorded in the keyway decreases with decreasing j which is due to the reduced pressure on the longitudinal side of the keyway (a same torsion moment transmitted). This is true as a design strategy. But stress are much higher than those obtained in the agreement. This states that the keyway is a more significant geometric discontinuity as stress concentrator, even coinciding with the area of the according radius between the two diameters.

Importantly, with respect to the previous study [17], the location of the maximum stress in the keyway provided in the fillet radius between the side and the bottom thereof, has moved from a position coincident with the curved end to the middle of the long keyway. This is because the criteria used to define the area under pressure, as the sector has been ignored in their inability curve for torque transmission.

The maximum stress recorded for the radius is located at approximately 90 ° to the keyway, and decreases as so does j. This, plus the fact unaffected by the proximity of the slot, is significantly different from the results obtained in the previous study [17]. The authors believe that this result is due to the bending moment generated by the lateral pressure in the keyway, significantly higher than the value of 1,000,000 Nm considered external load.

The approach to the effort applied torsion moment, a significant impact on the results. Considering the pressure of the key, to transmit the torque, on the longitudinal side of the slot is closer to the fact that when considering the loads applied as punctual.

Finally we can indicate that it is not possible to apply the criteria set by classical literature to obtain a stress concentration factor resulting from individuals for each geometric discontinuity and external stress.

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